Review Paper (T)

INVESTIGATION OF THERMAL CONDUCTIVITY AND VISCOSITY OF NANOFLUIDS

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ABSTRACT

A colloidal mixture of nano-sized (<100 nm) particles in a base liquid called nanofluid, which is the new generation of heat transfer fluid for various heat transfer applications where transport characteristics are substantially higher than the base liquid. This review summarizes and analyzes the empirical correlations for the effective thermal conductivity and dynamic viscosity of the nanofluid based on experimental data and theoretical model available in the literature. The review shows that the thermal conductivity ratio of the nanofluid to the base liquid for spherical and cylindrical nanoparticles increase appreciably with the increase of nanoparticle concentration and temperature. In addition, the viscosity ratio of the nanofluid to the base liquid also increase with the increase of nanoparticle concentration. This paper also surveys the mathematical models for estimation of thermal conductivity and viscosity of nanofluids and their limitations.

Key Words: Nanofluids, Thermal conductivity, Viscosity, Experimental, Model

Nomenclature:

\( c_p \) - Specific heat, J/kg-K
\( d_p \) - Diameter of nanoparticle, m
\( k \) - Thermal conductivity, W/m-K
\( k_{layer} \) - Thermal conductivity of layer formed between particle and fluid, W/m-K
\( Pr \) - Prandtl number of the base fluid
\( R \) - Coefficient of correlation
\( Re \) - Reynolds number
\( T \) - Temperature, K
\( T_f \) - Freezing point of the base fluid, K
\( t \) - Nano layer thickness, m

Greek symbols:

\( \alpha \) - Ratio of thermal conductivity of particle to that of base liquid
\( \gamma \) - Ratio of nanolayer thermal conductivity to particle thermal conductivity
\( \kappa \) - Boltzmann constant, 1.381x10^{-23} J/K
\( \mu \) - Dynamic viscosity, N-s/m²
\( \rho \) - Density, kg/m³
\( \phi \) - Volume fraction of nanoparticles
\( \psi \) - Shape factor

Subscripts:

\( eff \) - Effective
\( f \) - Base fluid
\( nf \) - Nanofluid
\( p \) - Nanoparticle
\( pe \) - Modified nanoparticle
\( r \) - Relative

INTRODUCTION

Industrial demands for high performance heat exchanger devices are increasing rapidly day by day. The poor physical properties of the conventional heat transfer fluids are the major problems in improving the performance of engineering equipment. The limited heat transfer capabilities of conventional heat transfer liquids such as water, ethylene glycol (EG), propylene glycol or engine oil have been overcome by the pioneer work of Ahuja¹ whereby the high thermal conductivity micro/millimeter-sized particles were suspended in the liquids. Heat transfer fluids containing suspended particles of micro/millimeter sizes have

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numerous drawbacks like sedimentation, erosion, fouling and increased pressure drop of the flow channel. Hence, they were not accepted as suitable material for next generation heat transfer medium. The recent advances in the material science have made it possible to produce nanometer-sizes particles that can overcome such problems. Innovative heat transfer fluids-suspended by nanometer-sized solid particles are called ‘nanofluids’. Nanofluids are the new generation heat transfer fluids for various industrial and automotive applications because of their excellent thermal performance (Eastman et al.\textsuperscript{2}). The main idea of dispersing solid nanoparticle in the base liquid has coined in 1995 by Choi\textsuperscript{3} at Argonne National Laboratory, USA which showed that the conventional liquid thermal performance could be remarkably improved using nanoparticles. Subsequent numerous investigations\textsuperscript{4-6} have shown that the nanofluids exhibited high thermal conductivity even for low concentrations of suspended nanoparticles. Nanofluids can be used for a wide variety of engineering applications like transportation, electronics, medical, food, defense, nuclear, space, and manufacturing of many types.\textsuperscript{7} Magnetic nanoparticles in bio-fluids can be used for medical applications as drug delivery vehicles, providing new cancer treatment technique. Using nanofluids in heat transfer applications will provide numerous other benefits including miniaturized heat exchangers, improved heat transfer, reduced heat transfer fluid inventory and reduced emissions. Use of nanofluids will conserve energy by reducing the necessary pumping power. These benefits make nanofluids a future generation heat transfer fluid. The suspended metallic and nonmetallic nanoparticles can change the transport and thermal properties of the base liquid. Among the transport properties, thermal conductivity and viscosity are the fundamental characteristics of fluid that influence the heat transfer phenomena.

AIMS AND OBJECTIVES

The aim of this paper is to present a critical review of the open literature and describe the generalized model for the thermal conductivity and viscosity of the nanofluid. To achieve this aim, we have identified several correlations from various research papers, which are based on different mechanisms of heat transport in nanofluids and established their validity against well documented practical results.

DISCUSSION

Thermal conductivity of nanofluids
Experimental investigations

One of the key feature for heat transfer enhancement is the thermal conductivity, the majority of the studies have discussed the thermal conductivity of nanofluids. All experimental results have pointed out the impressive improvement of thermal conductivity by employing nanoparticle. The equation proposed for the nanofluid effective thermal conductivity, normalized by the thermal conductivity of the base fluid, is derived from a wide variety of experimental data relative to nanofluids consisting of alumina, copper oxide and copper nanoparticles, suspended in water or ethylene glycol. These data are extracted from the following sources, Lee et al.\textsuperscript{4} for CuO + water, CuO + ethylene glycol, Al\textsubscript{2}O\textsubscript{3} + water, and Al\textsubscript{2}O\textsubscript{3} + ethylene glycol; Eastman et al.\textsuperscript{8} for Cu + ethylene glycol; Pak and Cho\textsuperscript{9} for TiO\textsubscript{2} + water; Das et al.\textsuperscript{10} for CuO + water, and Al\textsubscript{2}O\textsubscript{3} + water; Masuda et al.\textsuperscript{11} for TiO\textsubscript{2} + water; Chon et al.\textsuperscript{12} for Al\textsubscript{2}O\textsubscript{3} + water; Xuan et al.\textsuperscript{13} for TiO\textsubscript{2} + water; Murshed et al.\textsuperscript{14} for Al\textsubscript{2}O\textsubscript{3} + water, and Al\textsubscript{2}O\textsubscript{3} + ethylene glycol; Mintsa et al.\textsuperscript{15} for CuO + water; and Duangthongsuk and Wongwis\textsuperscript{16} for TiO\textsubscript{2} + water.

Analytical investigations

Earlier researchers like Maxwell\textsuperscript{17} Bruggeman et al.\textsuperscript{18} and Hamilton and crosser\textsuperscript{19} found out the correlations considering no effect of aggregation, interfacial layer and Brownian motion. The experimental data are much larger than the theoretical predictions according to the conventional models for the effective
conductivity of a solid/liquid suspension given by earlier researchers. To explain later researchers like Xie, Xue and Xuan found that the prediction of the earlier researchers holds good for very low concentration of nanofluid. Also some of the researchers found the empirical correlations by performing several numbers of experiments.

The Maxwell\(^{17}\) model was the first model to determine the thermal conductivity of liquid–solid suspensions. This model is applicable to statistically homogeneous and low-volume fraction liquid–solid suspensions with randomly dispersed, uniformly sized, and non-interacting spherical particles. The effective thermal conductivity, \(k_{\text{eff}}\) is given by:

\[
k_{\text{eff}} = \frac{k_p + 2k_f + 2(k_p - k_f)\phi}{k_p + 2k_f - (k_p - k_f)\phi}k_f
\]  

(1)

Bruggeman et al.\(^{18}\) proposed an implicit model for the effective thermal conductivity of solid–liquid mixtures, taking into account the interactions among the randomly distributed particles which is given by:

\[
\phi \left( \frac{k_p - k_{\text{eff}}}{k_p + 2k_{\text{eff}}} \right) + (1 - \phi) \left( \frac{k_f - k_{\text{eff}}}{k_f + 2k_{\text{eff}}} \right) = 1
\]

(2)

Bruggeman model can only be applied to spherical particles with no limitations on the particle volumetric concentrations.

Hamilton and Crosser\(^{19}\) extended the Maxwell’s model by introducing a shape factor to account for the effect of the shape of particles. The effective thermal conductivity of the solid/liquid mixture is given as follows:

\[
k_{\text{eff}} = \frac{k_p + (n-1)k_f - (n-1)(k_p - k_f)\phi}{k_p + (n-1)k_f + (k_f - k_p)\phi}k_f
\]

(3)

where, \(n\) is the empirical shape factor given by \(3/\psi\), and \(\psi\) is the particle sphericity which is defined as surface area of a sphere (with the same volume as the given particle) to the surface area of the particle. For spherical particle the value of \(n\) is 3.

Jeffrey\(^{20}\) correlation only considered the interactions among the randomly distributed particles. The correlation is given as follows

\[
k_{\text{eff}} = 1 + 3\beta\phi + \phi^2 \left( \frac{3\beta^2}{2} + \frac{9\beta^3}{4} + \frac{27\beta^4}{16} + \frac{3\beta^5}{4} + \cdots \right) k_f
\]

(4)

where \(\beta = \frac{a-1}{a+2}\)

Yu and Choi\(^{21}\) proposed a modified Maxwell model to include the effect of nanolayer surrounding the particles by removing ther thermal conductivity of solid particles with the equivalent thermal conductivity of particles \(k_{\text{pe}}\) can be expressed as follows

\[
k_{\text{pe}} = \frac{[2(1 - \gamma) + (1 + \beta)^2(1 + 2\gamma)]}{-(1 - \gamma) + (1 + \beta)^2(1 + 2\gamma)} k_p
\]

(6)

where \(\gamma = k_{\text{layer}} / k_p\) is the ratio of nanolayer thermal conductivity to particle thermal conductivity and \(\beta = h/r\) is the ratio of the nanolayer thickness to the particle radius. The nanolayer thickness \(h\) and the thermal conductivity \(k_{\text{layer}}\) are given to range from 1 to 2 nm and 2k\(_f\) < \(k_{\text{layer}}\) < 10k\(_f\), respectively. Finally the thermal conductivity of the nanofluid is given by

\[
k_{\text{nf}} = \frac{k_{\text{pe}} + 2k_f + 2(k_{\text{pe}} - k_f)(1 + \beta)^3\phi}{k_{\text{pe}} + 2k_f - (k_{\text{pe}} - k_f)(1 + \beta)^3\phi}k_f
\]

(7)

Xue et al.\(^{22}\) proposed a model considering the Brownian motion of nanoparticles and their aggregation. The modified correlation for the apparent thermal conductivity of nanofluid is the sum of the Maxwell’s model and the term due to Brownian motion of the nanoparticles and clusters. The resulting formula is

\[
k_{\text{nf}} = \frac{k_p + 2k_f - 2(k_p - k_f)\phi}{k_p + 2k_f + (k_p - k_f)\phi}k_f
\]

\[
+ \frac{\rho_p\psi_{\text{pp}}}{2} \frac{k_T}{3\pi r_c \mu_f}
\]

(8)

Here \(r_c\) is the mean radius of gyration of the cluster.

Xue and Xu\(^{23}\) developed an implicit relation for the effective thermal conductivity of copper oxide+ water and copper oxide+ ethylene glycol nanofluids based on a model of nanoparticles with interfacial shells between the surface of the solid particle and the surrounding liquid which is given by:
\[
\left(1 - \frac{\phi}{\alpha}\right) \times \frac{k_{nf} - k_f}{2k_{nf} + k_f} + \frac{\phi}{\omega} \left(\frac{k_{nf} - k_f}{2k_f + k_p} - \omega \left(\frac{k_p - k_f}{2k_f + k_p}\right)\right) = 0
\]
\[
(9)
\]

where \(\omega = \frac{r_p}{r_{p+1}}\)^3 \hspace{1cm} (10)

In the above equation \(k_f\) is the thermal conductivity of the interfacial shell and \(t\) represents the thickness of the interfacial shell, which are different for different nanofluids. The radius of the nanoparticle is \(r_p\).

Chon et al.\(^\text{12}\) proposed an empirical correlation for the thermal conductivity of H\(_2\)O nanofluid from their experimental data using Buckingham-Pi theorem with a linear regression scheme. They concluded that the Brownian motion of the suspended nanoparticle is the most important factor in the enhancement of thermal conductivity of nanofluids. The correlation is given as follows:

\[
k_{eff} = 1 + 64.7\phi^{0.7460} \left(\frac{d_f}{d_p}\right)^{0.3680} \left(\frac{k_p}{k_f}\right)^{0.7436}
\]
\hspace{1cm} (11)

Prasher et al.\(^\text{24}\) proposed that the enhancement in the thermal conductivity of nanofluids is primarily due to the convection caused by the Brownian motion of the nanoparticles. They introduced a convective-conductive model, which is a combination of Maxwell–Garnett conduction model and the convection caused by the Brownian motion of suspended nanoparticles can be expressed as:

\[
k_{eff} = \frac{1 + A Re^{m} Pr^{0.333} \phi}{\left(k_p(1 + 2\alpha + k_{m}) + 2\phi[k_p(1 - \alpha) - k_{m}]\right)}
\]
\hspace{1cm} (12)

where the coefficient \(A=4 \times 10^4\); \(m = 2.5 \pm (15\% \text{ of } 2.5)\) for water based nanofluids, \(m = 1.6 \pm (15\% \text{ of } 1.6)\) for ethylene glycol-based nanofluids and \(m = 1.05 \pm (15\% \text{ of } 1.05)\) for oil-based nanofluids; \(k_m = k_f [1 + (1/4) Re Pr]\) is the Brownian conductivity; \(Re = \frac{18 \pi T}{\nu^2}\) is the Brownian–Reynolds number; \(\alpha = 2R_{f}k_{m}/d_p\) is the nanoparticle Biot number; \(R_b\) is the interfacial thermal resistance between nanoparticles and different fluids; \(\nu\) is the kinematic viscosity and \(Pr\) is the Prandtl number of the base fluid.

Jang and Choi\(^\text{25}\) proposed a theoretical model that involves four modes contributing to the energy transfer resulting in enhancement of thermal conductivity of nanofluids. The first mode is collision between base fluid molecules, the second mode is the thermal diffusion in nanoparticles, the third mode is the collision of nanoparticles with each other due to the Brownian motion, and the fourth mode is collision between base fluid molecules and nanoparticles by thermally induced fluctuations. Including all the above four modes, the effective thermal conductivity of nanofluid is given as follows:

\[
k_{nf} = k_f(1 - \phi) + \beta_1 k_p \phi + C_1 \frac{d_f}{d_p} Re_{d_p}^2 Pr \phi
\]
\hspace{1cm} (13)

where \(\beta_1 = 0.01\) is a constant for considering the Kapitza resistance per unit area; \(C_1 = 18 \times 10^6\) is a proportionality constant; \(Pr\) is the Prandtl number of the base fluid, and the Reynolds number is defined by \(Re_{d_p} = \frac{C_{R,M} d_p}{\nu}\), where \(C_{R,M} = \frac{k T}{3 \pi \mu_f d_p l_f}\) is the random motion velocity of a nanoparticle and \(\nu\) is the kinematic viscosity of base fluid. They recommend, for water-based nanofluids the equivalent diameter \(d_f=0.384\) nm and mean-free path \(l_f = 0.738\) nm at a temperature of 300 K.

Massimo Corcione\(^\text{26}\) proposed empirical correlation for predicting the effective thermal conductivity based on a high number of experimental data available in the literature. It is found that, for given the nanoparticle material and the base fluid, the ratio between the thermal conductivities of the nanofluid and the pure base liquid increases as the nanoparticle volume fraction and the temperature are increased, and the nanoparticle diameter is decreased. The ease of application of the equations proposed, and their wide regions of validity (the ranges of the nanoparticle diameter, volume fraction and temperature are 10–150 nm, 0.002–0.09 and 294–324 K) for the thermal conductivity data, make such equations useful by the engineering point of view, for both numerical simulation purposes and thermal design tasks.
Different correlations have been compared on the basis of the experiment, performed by Corcione²⁶

\[
\frac{k_{\text{eff}}}{k_f} = 1 + 4.4 \text{Re}^{0.4} \text{Pr}^{0.66} \left(\frac{T}{T_{fr}}\right)^{10} \left(\frac{k_p}{k_f}\right)^{0.03} \phi^{0.66}
\]  

(14)

Al₂O₃ (44 nm) + water at 324 K temperature and 1 atm pressure. Further the different parameters that were used for different correlations are given in Table 1 and the ratio of thermal conductivity of nanofluid and base fluid at different concentration are presented in Table 2.

**Table 1 : Different parameters of nanofluid and base liquid**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (T)</td>
<td>324K</td>
</tr>
<tr>
<td>Pressure (P)</td>
<td>1 atm</td>
</tr>
<tr>
<td>Thermal conductivity of base fluid, k&lt;sub&gt;f&lt;/sub&gt;</td>
<td>0.6316 [W/m-K]</td>
</tr>
<tr>
<td>Thermal conductivity of nano particle, k&lt;sub&gt;p&lt;/sub&gt;</td>
<td>40 [W/m-K]</td>
</tr>
<tr>
<td>Thermal conductivity of interfacial layer, k&lt;sub&gt;layer&lt;/sub&gt;</td>
<td>3.2 [W/m-K]</td>
</tr>
<tr>
<td>Specific heat of base fluid, c&lt;sub&gt;pf&lt;/sub&gt;</td>
<td>4.181 [kJ/kg-K]</td>
</tr>
<tr>
<td>Specific heat of nano particle, c&lt;sub&gt;pp&lt;/sub&gt;</td>
<td>0.765 [kJ/kg-K]</td>
</tr>
<tr>
<td>Density of base fluid, ρ&lt;sub&gt;f&lt;/sub&gt;</td>
<td>987.6 [kg/m³]</td>
</tr>
<tr>
<td>Density of nano particle, ρ&lt;sub&gt;p&lt;/sub&gt;</td>
<td>3600 [kg/m³]</td>
</tr>
<tr>
<td>Viscosity of base fluid, μ&lt;sub&gt;f&lt;/sub&gt;</td>
<td>0.000538 [kg/m-s]</td>
</tr>
<tr>
<td>Diameter of nano particle, d&lt;sub&gt;p&lt;/sub&gt;</td>
<td>44 nm</td>
</tr>
<tr>
<td>Interfacial thickness or nano layer thickness, t</td>
<td>2 nm</td>
</tr>
<tr>
<td>Packing fraction of water, m</td>
<td>2.5</td>
</tr>
<tr>
<td>Maximum particle concentration, Φ&lt;sub&gt;m&lt;/sub&gt;</td>
<td>5.5</td>
</tr>
</tbody>
</table>

**Table 2 : The ratio of thermal conductivity of nanofluid and base fluid using different model at different concentration**

<table>
<thead>
<tr>
<th>Concentration</th>
<th>Maxwe²⁷</th>
<th>Brugge²⁸</th>
<th>Hamilton and Crosser²⁹</th>
<th>Jeffrey³⁰</th>
<th>Yu and Choi³¹</th>
<th>Xuan et al.³²</th>
<th>Xue and Xu³³</th>
<th>Chon et al.³⁴</th>
<th>Prashe²⁸</th>
<th>Jang and Choi³⁵</th>
<th>Corcione³⁶</th>
<th>Experimental</th>
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<tbody>
<tr>
<td>0.005</td>
<td>1.014</td>
<td>1.015</td>
<td>1.014</td>
<td>1.014</td>
<td>1.021</td>
<td>1.036</td>
<td>1.022</td>
<td>1.077</td>
<td>1.063</td>
<td>1.028</td>
<td>1.091</td>
<td>1.09</td>
</tr>
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<td>0.01</td>
<td>1.029</td>
<td>1.03</td>
<td>1.029</td>
<td>1.029</td>
<td>1.043</td>
<td>1.072</td>
<td>1.046</td>
<td>1.128</td>
<td>1.132</td>
<td>1.056</td>
<td>1.144</td>
<td>1.145</td>
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<tr>
<td>0.015</td>
<td>1.044</td>
<td>1.045</td>
<td>1.044</td>
<td>1.044</td>
<td>1.064</td>
<td>1.108</td>
<td>1.07</td>
<td>1.174</td>
<td>1.202</td>
<td>1.084</td>
<td>1.188</td>
<td>1.185</td>
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<td>0.02</td>
<td>1.059</td>
<td>1.061</td>
<td>1.059</td>
<td>1.059</td>
<td>1.086</td>
<td>1.144</td>
<td>1.096</td>
<td>1.215</td>
<td>1.271</td>
<td>1.112</td>
<td>1.227</td>
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<td>1.077</td>
<td>1.074</td>
<td>1.074</td>
<td>1.109</td>
<td>1.181</td>
<td>1.122</td>
<td>1.254</td>
<td>1.341</td>
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<tr>
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<td>1.094</td>
<td>1.089</td>
<td>1.089</td>
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<td>1.217</td>
<td>1.15</td>
<td>1.292</td>
<td>1.41</td>
<td>1.168</td>
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<tr>
<td>0.035</td>
<td>1.104</td>
<td>1.111</td>
<td>1.105</td>
<td>1.105</td>
<td>1.154</td>
<td>1.254</td>
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<td>1.119</td>
<td>1.121</td>
<td>1.121</td>
<td>1.291</td>
<td>1.209</td>
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<td>1.551</td>
<td>1.224</td>
<td>1.358</td>
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<tr>
<td>0.045</td>
<td>1.135</td>
<td>1.148</td>
<td>1.135</td>
<td>1.137</td>
<td>1.201</td>
<td>1.328</td>
<td>1.24</td>
<td>1.394</td>
<td>1.621</td>
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<td>0.05</td>
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<td>1.167</td>
<td>1.151</td>
<td>1.153</td>
<td>1.225</td>
<td>1.365</td>
<td>1.273</td>
<td>1.427</td>
<td>1.691</td>
<td>1.28</td>
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<tr>
<td>0.055</td>
<td>1.167</td>
<td>1.186</td>
<td>1.167</td>
<td>1.169</td>
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<td>1.402</td>
<td>1.308</td>
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<td>1.308</td>
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<td>1.183</td>
<td>1.185</td>
<td>1.275</td>
<td>1.44</td>
<td>1.337</td>
<td>1.489</td>
<td>1.833</td>
<td>1.356</td>
<td>1.468</td>
<td>1.47</td>
</tr>
</tbody>
</table>
As one can see from the Table 3 that the first three classical models (Maxwell\textsuperscript{17}, Bruggeman et al.\textsuperscript{18}, and Hamilton-Crosser\textsuperscript{19}) under predicts the thermal conductivity of nanofluids for all values of particle volume concentration. The Chon et al.\textsuperscript{12} model and Corcione\textsuperscript{26} model gives reasonably accurate results, but both of these models being empirical correlations, have their limitations. While Chon et al.\textsuperscript{12} model is only valid for Al\textsubscript{2}O\textsubscript{3} based nanofluids, Corcione\textsuperscript{26} model fails to take into account the agglomeration of nanoparticles. The Prasher et al.\textsuperscript{24} model gives accurate results for low particle volume concentrations (upto about 1.5%) but over predicts the thermal conductivity for higher values. The Xuan et al.\textsuperscript{22} model predicts closer to experimental results for

\[ \text{Table 3: The percentage deviations using different model of the ratio — from Corcione's experiment at different concentrations} \]

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
Concentration & Maxwell\textsuperscript{17} & Bruggeman et al.\textsuperscript{18} & Hamilton and Crosser\textsuperscript{19} & Jeffry\textsuperscript{20} & Yu and Choi\textsuperscript{21} & Xuan et al.\textsuperscript{22} & Xue and Xu\textsuperscript{23} & Chon et al.\textsuperscript{12} & Prasher et al.\textsuperscript{24} & Jang and Choi\textsuperscript{25} & Corcione\textsuperscript{26} \\
\hline
0.005 & 6.42 & 6.33 & 6.42 & 6.33 & 5.78 & 4.59 & 5.69 & 1.10 & 2.29 & 5.23 & 0.09 \\
0.01 & 8.82 & 8.73 & 8.82 & 8.82 & 7.77 & 5.59 & 7.51 & 1.31 & 0.96 & 6.81 & 0.09 \\
0.015 & 10.04 & 9.96 & 10.04 & 10.04 & 8.61 & 5.49 & 8.19 & 0.76 & 1.18 & 7.17 & 0.25 \\
0.02 & 11.10 & 10.94 & 11.10 & 11.10 & 9.22 & 5.39 & 8.57 & 0.65 & 3.10 & 7.51 & 0.16 \\
0.025 & 11.94 & 11.78 & 11.94 & 11.94 & 9.72 & 5.22 & 8.93 & 0.71 & 4.74 & 7.83 & 0.16 \\
0.03 & 12.28 & 11.97 & 12.28 & 12.28 & 9.81 & 4.63 & 8.65 & 0.15 & 6.87 & 7.57 & 0.08 \\
0.035 & 12.78 & 12.41 & 12.78 & 12.71 & 9.92 & 4.29 & 8.57 & 0.15 & 8.50 & 7.59 & 0.15 \\
0.04 & 13.01 & 12.50 & 13.01 & 12.94 & 9.85 & 3.75 & 8.16 & 0.07 & 10.29 & 7.35 & 0.07 \\
0.045 & 13.17 & 12.52 & 13.17 & 13.09 & 9.78 & 3.24 & 7.77 & 0.22 & 11.94 & 7.12 & 0.14 \\
0.05 & 13.31 & 12.54 & 13.31 & 13.24 & 9.65 & 2.75 & 7.32 & 0.35 & 13.45 & 6.97 & 0.28 \\
0.055 & 13.29 & 12.39 & 13.29 & 13.22 & 9.34 & 2.08 & 6.57 & 0.62 & 15.16 & 6.57 & 0.14 \\
0.06 & 13.27 & 12.24 & 13.27 & 13.20 & 9.05 & 1.36 & 6.12 & 0.88 & 16.80 & 6.19 & 0.07 \\
\hline
\end{tabular}
higher values of particle volume concentration (above 5%).

**Investigation for the effective dynamic viscosity**

Many experimental and theoretical works have been dedicated to the thermal conductivity of nanofluids. However, very few experimental studies have been devoted to the nanofluid viscosity. 5,9,11,14,26-28

To our knowledge there is no exact theoretical modeling for nanofluid viscosity. However, a few theoretical models have been presented for the determination of a particle suspension viscosity. The Einstein model29 is commonly used to predict the effective viscosity of suspensions containing a low volume fraction of particles (usually <0.01). His formula of effective thermal conductivity is

\[
\mu_r = \frac{\mu_{eff}}{\mu_f} = 1 + 2.5\phi
\]  

(15)

Brinkman30 generalized the Einstein correlation for higher concentrations:

\[
\frac{\mu_{eff}}{\mu_f} = \frac{1}{(1 - \phi)^{2.5}}
\]  

(16)

For simple hard sphere systems, the relative viscosity increases with particle volume fraction \(\phi\). To determine this relative viscosity, a semi-empirical equation formulated by Krieger and Dougherty31 expressed as follows

\[
\frac{\mu_{eff}}{\mu_f} = \left(1 - \frac{\phi}{\phi_m}\right)^{-[\eta]/\phi_m}
\]  

(17)

where \(\phi_m\) is the maximum packing fraction and \([\eta]\) is the intrinsic viscosity ([\(\eta\] = 2.5 for hard spheres). For randomly mono-dispersed spheres, the maximum close packing fraction is approximately 0.64.31

A decade later, a generalized equation for the relative viscosity was proposed by Nielsen32

For a concentration of dispersed particles, Nielsen’s equation can be simplified as follows

\[
\frac{\mu_{eff}}{\mu_f} = (1 + 1.5\phi)
\]

(18)

where \(\phi\) and \(\phi_m\) are the volume fraction of particles and the maximum packing fraction, respectively.

Batchelor33 considered the effect of the Brownian motion of rigid and spherical particles and developed the following correlation

\[
\frac{\mu_{eff}}{\mu_f} = 1 + 2.5\phi + 6.5\phi^2
\]  

(19)

Graham34 generalized the Frankel and Acrivos work. The correlation was presented for low concentrations like the one developed by Einstein29

\[
\frac{\mu_{eff}}{\mu_f} = 1 + 2.5\phi
\]

(20)

+ 4.5 \left[\frac{1}{\left(\frac{h}{d_p}\right)^2}\left(1 + \frac{h}{d_p}\right)^2\right]

Massimo Corcione26 proposed empirical correlation for predicting the relative viscosity,

\[
\frac{\mu_{eff}}{\mu_f} = \frac{1}{1 - 34.87(\frac{d_p}{d_f})^{-0.3}}\phi^{1.03}
\]  

(21)

where \(d_f\) is the equivalent diameter of a base fluid molecule, given by

\[
d_f = .1 \left(\frac{6M}{N\pi\rho_{fo}}\right)^{1/3}
\]  

(22)

in which \(M\) is the molecular weight of the base liquid, \(N\) is the Avogadro number, and \(\rho_{fo}\) is the mass density of the base liquid calculated at temperature \(T_0 = 293\text{K}\). The experiment on viscosity measurement was performed by Nguyen et al.28 for Al2O3+ water nanofluid. The comparison of test data with predicted data from various correlations is shown in Table 4 and Fig. 2.

It can be seen from Table 5, that for lower values (0.005-0.01) of particle volume concentration, most of the models gives results closer to experimental values. For intermediate values of particle volume concentration (1-2%), Nielsen model gives better results. None of the existing models explains the enhanced viscosity for higher values of particle volume concentration.
Table 4: Relative viscosity using different model at different concentrations

<table>
<thead>
<tr>
<th>Concentration</th>
<th>Einstein$^{29}$</th>
<th>Brinkmann$^{30}$</th>
<th>Krieger$^{31}$</th>
<th>Nielsen$^{32}$</th>
<th>Batchelor$^{33}$</th>
<th>Graham$^{34}$</th>
<th>Corcione$^{26}$</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>1.013</td>
<td>1.013</td>
<td>1.013</td>
<td>1.022</td>
<td>1.013</td>
<td>1.036</td>
<td>1.04</td>
<td>1.016</td>
</tr>
<tr>
<td>0.01</td>
<td>1.025</td>
<td>1.025</td>
<td>1.026</td>
<td>1.044</td>
<td>1.026</td>
<td>1.049</td>
<td>1.084</td>
<td>1.04</td>
</tr>
<tr>
<td>0.015</td>
<td>1.038</td>
<td>1.039</td>
<td>1.039</td>
<td>1.066</td>
<td>1.039</td>
<td>1.061</td>
<td>1.134</td>
<td>1.071</td>
</tr>
<tr>
<td>0.02</td>
<td>1.05</td>
<td>1.052</td>
<td>1.052</td>
<td>1.089</td>
<td>1.053</td>
<td>1.074</td>
<td>1.189</td>
<td>1.11</td>
</tr>
<tr>
<td>0.025</td>
<td>1.063</td>
<td>1.065</td>
<td>1.066</td>
<td>1.112</td>
<td>1.067</td>
<td>1.086</td>
<td>1.25</td>
<td>1.156</td>
</tr>
<tr>
<td>0.03</td>
<td>1.075</td>
<td>1.079</td>
<td>1.08</td>
<td>1.136</td>
<td>1.081</td>
<td>1.099</td>
<td>1.318</td>
<td>1.21</td>
</tr>
<tr>
<td>0.035</td>
<td>1.087</td>
<td>1.093</td>
<td>1.094</td>
<td>1.16</td>
<td>1.095</td>
<td>1.111</td>
<td>1.394</td>
<td>1.271</td>
</tr>
<tr>
<td>0.04</td>
<td>1.1</td>
<td>1.107</td>
<td>1.109</td>
<td>1.185</td>
<td>1.11</td>
<td>1.124</td>
<td>1.48</td>
<td>1.34</td>
</tr>
</tbody>
</table>

Fig. 2: Variation of relative viscosity using different model at different concentrations

Table 5: The percentage deviations using different model of the relative viscosity (---) from Nguyen et al.$^{28}$ experiment at different concentration

<table>
<thead>
<tr>
<th>Concentration</th>
<th>Einstein$^{29}$</th>
<th>Brinkmann$^{30}$</th>
<th>Krieger$^{31}$</th>
<th>Nielsen$^{32}$</th>
<th>Batchelor$^{33}$</th>
<th>Graham$^{34}$</th>
<th>Corcione$^{26}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.59</td>
<td>0.30</td>
<td>1.97</td>
<td>2.36</td>
</tr>
<tr>
<td>0.01</td>
<td>1.44</td>
<td>1.44</td>
<td>1.35</td>
<td>0.38</td>
<td>1.35</td>
<td>0.87</td>
<td>4.23</td>
</tr>
<tr>
<td>0.015</td>
<td>3.08</td>
<td>2.99</td>
<td>2.99</td>
<td>0.47</td>
<td>2.99</td>
<td>0.93</td>
<td>5.88</td>
</tr>
<tr>
<td>0.02</td>
<td>5.41</td>
<td>5.23</td>
<td>5.23</td>
<td>1.89</td>
<td>5.14</td>
<td>3.24</td>
<td>7.12</td>
</tr>
<tr>
<td>0.025</td>
<td>8.04</td>
<td>7.87</td>
<td>7.79</td>
<td>3.81</td>
<td>7.70</td>
<td>6.06</td>
<td>8.13</td>
</tr>
<tr>
<td>0.03</td>
<td>11.16</td>
<td>10.83</td>
<td>10.74</td>
<td>6.12</td>
<td>10.66</td>
<td>9.17</td>
<td>8.93</td>
</tr>
<tr>
<td>0.035</td>
<td>14.48</td>
<td>14.00</td>
<td>13.93</td>
<td>8.73</td>
<td>13.85</td>
<td>12.59</td>
<td>9.68</td>
</tr>
<tr>
<td>0.04</td>
<td>17.91</td>
<td>17.39</td>
<td>17.24</td>
<td>11.57</td>
<td>17.16</td>
<td>16.12</td>
<td>10.45</td>
</tr>
</tbody>
</table>

CONCLUSION
A systematic survey of effective thermal conductivity and dynamic viscosity of nanofluids has been presented based on large number models, experimental correlations available in the existing literature. It has been observed that, the thermal conductivity ratio of the nanofluid to the base liquid increase appreciably at high nanoparticle concentration and temperature. Moreover, the dynamic viscosity ratio of the nanofluid to the base liquid also increase with the increase of nanoparticle volume fraction and decrease of...
the nanoparticle diameter, but surprisingly independent of temperature. Researchers have proposed many models and performed numerous experiments for characterization of thermophysical properties of nanofluid. It has been observed that, many existing thermal conductivity models of nanofluids do not agree well with the present experimental results. It is unclear up till now, which is the suitable model to use for determination of thermal conductivity of nanofluids as different conductivity prediction models are based on different transport mechanisms. Therefore further study is required to investigate on transport mechanism for prediction of an unique effective thermal conductivity and dynamic viscosity related to the nanofluid. However, we have compared different theoretical transport model using experimental data obtained by Coricone among which Coricone’s experimental data based correlation gives accurate prediction of thermal conductivity and viscosity of different nanofluids and found best agreement with the experimental results.

REFERENCES