RIVER FLOW MODEL USING LINK NODE SCHEME

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ABSTRACT

The river flow models are the basic model for any pollutant transport and flood simulation in a river reach. Link node networks forms the basis for most of the river flow models. This complete-one dimensional hydrodynamic model simulates the dynamics of the water movements in the river and calculates the flow characteristics at each cross section during given time step. Due to unsteady flow in a given river reach, unsteady continuity equation and the complete solution methods are desirable to solve the one-dimensional momentum equation. In the present study an explicit finite difference numerical model has been developed using link node scheme with the objective of predicting the water surface elevation and velocity distributions simultaneously across the entire model domain. The developed model is then applied to Androscoggin river, USA and validated with the measured depth and velocity. The objective is accomplished by applying the one-dimensional equation of motion to the river reach to predict river velocity and the continuity equation at junctions to predict fluctuations in the water surface elevation. It is found that the simulated and observed depth coincides and there is no variation. However the variations in the simulated and observed velocity shows the unstable nature of explicit finite difference method, especially for higher time step and indicates that other numerical modeling techniques like implicit finite difference or finite element method is required to compute the velocity variation for unsteady flow.

Key Words : River flow, Link node scheme, Unsteady flow methods, Complete dynamic equation, Finite difference scheme

INTRODUCTION

Generally flow in a river reach is of unsteady in nature and rarely steady state flow exists. Unsteady flow leads to spatial and temporal variation in depth and velocity. Number of explicit schemes has been used to solve the set of Saint-Venant equations. During the past two decades much progress has been made in the use of numerical models for simulation of the hydrodynamics of surface water flows. Naidu et al.1 developed an iterative method for computing the water surface profile in a tree-type channel networks. This method solves for the water surface profile in a single source binary tree type of network, given the inflow discharge at the root and the control depths at all of the downstream ends. Reddy and Bhallamudi2 formulated an iterative method for computing the water surface profiles in cyclic looped channel networks. The method determines the water surface profile in a network given the discharge in the inflow channel and the control depth at the downstream point. Chagas and Souza3 developed a numerical model to study the flood wave propagation with an explicit solution. Islam et al.4 solved the Saint–Venant equations using the four point implicit finite-difference scheme for the solution of gradually varied flow in open-channel networks and the Newton–Raphson method used for getting solution of discretized nonlinear equations. Akbari and Firoozi5 used Preissmann and Lax diffusive schemes for the numerical solution of Saint-Venant equations that govern the propagation of flood wave, in natural rivers. The results of these numerical solutions are compared with the HEC-RAS. Litrico et al.6 developed a simple nonlinear model representative of the flow transfer in a river reach using diffusive wave equation and the Saint-Venant equations. Moghaddam and Firoozi7,8 developed Preissmann implicit finite difference scheme for unsteady flow in open channels for full Saint-Venant equations.
The comparison of the numerical solution results with the HEC- RAS model does not match with each other. However the link node networks are the basis for most of the river flow models. The present study illustrates the explicit solution of the complete one dimensional saint–venant equations in link-node model network using finite difference technique.

**Study area**

The lower Androscoggin river watershed extends from New Hampshire, New England to Maine USA and contains the Androscoggin, Little Androscoggin and Sabattus river. The study area of length 35.5 km river stretch, extends from the Auburn to Brunswick, Maine. The river passes through the twin cities of Lewiston and Auburn, Maine turns southeast, passes the community of Lisbon Falls and reaches tidewater just below the final falls in the town of Brunswick, USA Maine. Merrymeeting Bay USA is a 16 km long freshwater estuary where the Androscoggin meets the Kennebec river, USA nearly 20 miles (32 km) inland from the Atlantic Ocean (Fig. 1).

The selected Androscoggin river reach is divided into 12 segments (**Table 1**) depending upon the variation in width and depth. Thus the river reach consist of 13 junctions and 12 segments. Normally junctions are referred to nodes and segments as the links which connects the nodes. This data along with the hourly observed discharge has been used in the link node scheme using explicit finite difference technique.

![Fig 1: The study area of the Androscoggin river reach](image)

**Table 1 : River segment data**

<table>
<thead>
<tr>
<th>Segment</th>
<th>Segment Name</th>
<th>River - km Begin</th>
<th>Length (m)</th>
<th>Width (m)</th>
<th>Depth (m)</th>
<th>Surface Elevation (m)</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Little Androscoggin R. to Dresser’s rapids (Auburn)</td>
<td>35.56</td>
<td>2092.14</td>
<td>208.38</td>
<td>2.09</td>
<td>20.35</td>
<td>0.0072</td>
</tr>
<tr>
<td>2</td>
<td>Dresser’s rapids</td>
<td>33.48</td>
<td>965.61</td>
<td>132.00</td>
<td>2.62</td>
<td>20.20</td>
<td>0.3093</td>
</tr>
<tr>
<td>3</td>
<td>Dresser’s rapids to Durham boat ramp</td>
<td>32.51</td>
<td>7724.85</td>
<td>161.63</td>
<td>3.10</td>
<td>17.20</td>
<td>0.0188</td>
</tr>
<tr>
<td>4</td>
<td>Durham boat ramp to Island</td>
<td>24.78</td>
<td>3540.56</td>
<td>234.42</td>
<td>2.11</td>
<td>15.75</td>
<td>0.0946</td>
</tr>
<tr>
<td>5</td>
<td>Island to sabattus rapids</td>
<td>21.24</td>
<td>4345.23</td>
<td>4345.23</td>
<td>2.19</td>
<td>12.4</td>
<td>0.0530</td>
</tr>
<tr>
<td>6</td>
<td>Sabattus rapids</td>
<td>16.90</td>
<td>1287.48</td>
<td>143.06</td>
<td>3.64</td>
<td>7.85</td>
<td>0.2442</td>
</tr>
<tr>
<td>7</td>
<td>Worumbo dam impoundment</td>
<td>16.42</td>
<td>3379.62</td>
<td>249.23</td>
<td>3.60</td>
<td>8.85</td>
<td>0.0296</td>
</tr>
<tr>
<td>8</td>
<td>Little river segment</td>
<td>13.04</td>
<td>1287.48</td>
<td>143.06</td>
<td>3.64</td>
<td>7.85</td>
<td>0.2442</td>
</tr>
<tr>
<td>9</td>
<td>Pejepscot dam impoundment</td>
<td>11.75</td>
<td>4184.29</td>
<td>175.94</td>
<td>3.95</td>
<td>4.70</td>
<td>0.0036</td>
</tr>
<tr>
<td>10</td>
<td>Topsham dam impoundment US reach</td>
<td>7.56</td>
<td>1287.48</td>
<td>124.33</td>
<td>3.39</td>
<td>4.55</td>
<td>0.3047</td>
</tr>
<tr>
<td>11</td>
<td>Topsham dam impoundment main reach</td>
<td>6.28</td>
<td>5310.83</td>
<td>205.13</td>
<td>3.43</td>
<td>0.65</td>
<td>0.0104</td>
</tr>
<tr>
<td>12</td>
<td>Brunswick</td>
<td>0.97</td>
<td>804.70</td>
<td>184.76</td>
<td>6.20</td>
<td>0.10</td>
<td>0.0123</td>
</tr>
</tbody>
</table>
MATERIAL AND METHODS

The main objective of a numerical simulation is to transform a complex real-life problem into a simple discrete form of mathematical description to recreate the system, solve it and finally reveal the phenomena virtually according to the requirements of the analysts. The Saint–Venant equations solved for the junction and channel are the continuity and momentum equation respectively.

The equation of continuity – conservation of mass

A junction is a volumetric unit that conceptually serves as a receptacle for water. The junction receives water from the channels with which it is connected. The equation solved at the junctions is the continuity equation.

\[
\frac{\partial Y}{\partial t} = -\frac{1}{W} \frac{\partial Q}{\partial x} \tag{1}
\]

The continuity equation in finite difference form

The continuity equation (Eq.1) at the \(\zeta\) junction can be written in finite difference form, using a forward difference for a time derivative \(\Delta t\) as:

\[
\frac{Y_{\zeta,j} - Y_{\zeta,j-1}}{\Delta t} = -\frac{\sum Q_{\zeta}}{W \Delta x_{\zeta}} \tag{2}
\]

Where,

\(Y\) = the mean depth in m

\(W\) = the mean channel width in m

\(Q\) = flow in \(m^3/s\)

\(Y_{\zeta,j}\) = the average depth of flow associated with the junction \(\zeta\) at time \(t\)

\(Y_{\zeta,j-1}\) = the average depth at the previous time step

\(Q_{\zeta}\) = flows entering a junction from channels

\(W_{\zeta}\) = width of the junction element \(\zeta\)

\(\Delta x_{\zeta}\) = the length of a junction element \(\zeta\)

\(A_{\zeta}^S\) = the surface area of a junction element in \(m^2\)

The equation of motion – conservation of momentum

A channel conveys water between junctions. The equation solved for the channel is the momentum equation.

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} = -\frac{g n^2}{\delta^2 R^3} \left[ U^2 - g \frac{\partial h}{\partial x} \right] \tag{3}
\]

The momentum equations in finite difference form

The equation of motion (Eq.3) can be written in finite difference form, using a forward difference for a time derivative \(\Delta t\) as:

\[
\frac{U_{j,t} - U_{j,t-1}}{\Delta t} = -U_{j,t-1} \left( \frac{\Delta U_j}{\Delta x_j} - \frac{g n^2}{\delta^2 R^3} U_{j,t-1} \right) \tag{4}
\]

\[
\frac{\partial U_j}{\partial x_j} = \frac{W_j}{A_j} \frac{\partial Y_j}{\partial t} - \frac{U_j}{A_j} \frac{\partial A_j}{\partial x_j} \tag{5}
\]

Where,

\(U\) = longitudinal velocity in m/sec

\(\Delta x\) = distance along the longitudinal axis (L) in m

\(g\) = acceleration of gravity in m/sec

\(n\) = manning roughness coefficient

\(\delta\) = unit conversion factor

\(U_{j,t}\) = the average velocity in channel \(j\) at time \(t\) in m/sec

\(U_{j,t+1}\) = the average velocity at the end of the previous time step in m/sec

\(\Delta t\) = the time step in sec

\(\Delta U_j\) = the change in velocity over the length of channel \(j\) between junctions \(\zeta\) and \(\zeta + 1\).

\(R\) = the hydraulic radius of the channel \(j\)

\(\Delta h_j\) = the change in water surface elevation over the length of channel \(j\)

\(A_j\) = the cross-sectional area of channel \(j\)

\(\Delta Y_j\) = the change in depth over the channel \(j\) between junctions \(\zeta\) and \(\zeta + 1\)

\(\Delta A_j\) = the change in cross-sectional area over the length of channel \(j\) between junctions \(\zeta\) and \(\zeta + 1\)

\(\Delta L_j\) = the average of the changes in the head at the two junctions during the time step \(\Delta t\).

\(\Delta A_j/\Delta x_j\) = the difference in cross-sectional areas of the two junctions divided by the channel length.

RESULTS AND DISCUSSION

In the present work, the above developed Saint-Venant equations are solved for a river length of 35.5 km using finite difference explicit scheme. The variations of flow and velocity along the river reach are computed using a link-node scheme with
13 junctions and 12 segments. Based on the cross sectional variation, segment length is selected. For the known river flow along 13 junctions, flow depth in 13 junctions and the velocity along 12 segments are determined. The river flow was measured on 2/8/2010 along Androscoggin river at 5 locations namely, Monty, Auburn, Little river confluence, Sabattus river confluence and Brunswick. The river flow measurements are done at every one hour interval for 24 hours on 2/8/2010. Each discharge measurement locations are considered to be junction or nodes and the links are connecting the two junctions. Through the link-node model river flow depth is calculated at every junction and velocity at the channels between the junctions. Since, the flow in the river is unsteady, there will be spatial and temporal variations in depth and velocity.

The flow properties pertaining to the unsteady flow in the river reach is arrived by solving simultaneously the continuity and momentum equations. In the present work, link-node scheme is used to predict the river flow depth and velocity at time step of 5 seconds. This is accomplished by applying the one-dimensional equation of motion to the network links to predict river flow depth and applying the continuity equation to the network nodes to predict fluctuations in the water surface elevation. The variations of depth over the period of 24 hours in each junction are shown in the Fig. 2. From it is

Fig 2 : Depth of flow at various junctions in 24 hours time

![Fig 2](image_url)

Fig 3 : Depth of flow at Junction 1

![Fig 3](image_url)
seen that there is large variation in the depth of flow among the junctions. This is due to variation in the width of the channel.

A closer look at the depth of flow variation at junction 1 over 24 hours in Fig. 3 shows that the flow is unsteady. Hence this unsteady flow has been used to predict the depth of flow using Saint-Venant equation and the resulted depth along with observed depth at various junctions is shown in the Fig. 4. The simulated and observed depth graph coincides with each other and there is no variation in all 24 hours (Fig.5). This is due to the absence of lateral inflow into the river junctions. Moreover in link-node scheme river flow depth can be estimated only at the junctions, which is of continuity as per the
Fig 5: Comparison of observed and simulated velocity governing equation. Fig. 5 depicts the hourly velocity variations at different segments and Fig.6 shows the comparison of observed and simulated velocity on various reaches. In the present study area, one can notice that the velocity varies largely in the segments where the length is smaller and the smaller velocity variation in the segments where the length is maximum. This can be seen very clearly in the segment 2 and segment 6 in the Fig.6. Hence, this present study indicates that the physical parameters of the river reach play the major role in the velocity variations.

CONCLUSION

The explicit solution of the complete one dimensional saint-venant equations in link-node model network using finite difference technique was developed, solved and analyzed in the present study. The developed model is applied to Androscoggin River, USA and verified using the measured depth and velocity over 24 hours at 1 hour interval. The simulated and observed depth coincides and there is no variation in all 24 hours. The simulated and observed velocity are compared and found out that there is some variation in the second and sixth segment, where the slope is larger and length is less as compared to the other segments. The variations in the simulated and observed velocity show the unstable nature of explicit finite difference method, especially for higher time step. In the explicit scheme at the higher time step, the observed velocity mismatches with the numerical solution. This analysis indicates that the overlapping network scheme results in reasonable result, if the selected segment length is large and time step is small. Smaller length for larger slope in the link node scheme gives the erroneous results. Thus, link node scheme is an abstract method and can be used only for preliminary velocity and depth calculation.

REFERENCES


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